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SOMBOR INDEX OF SOME GRAPH VALUED FUNCTIONS OF SUBDIVISION GRAPHS

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Abstract: The Sombor index $SO(G)$ has recently introduced in the chemical graph theory. It is a vertex-degree-based topological index defined as $SO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}$, where $d_G(u)$ is the degree of a vertex u in a graph G. In this paper we obtain Sombor index of line graph, total graph, semitotal line graph and semitotal point graph of subdivision graph.

Keywords and Phrases: Sombor index, subdivision graph, total graph, semitotal line graph, semitotal point graph.

2020 Mathematics Subject Classification: 05C09, 05C92.

1. Introduction

A topological index is a kind of molecular descriptor that is computed for the molecular graph of a chemical compound in the fields of chemical graph theory, molecular topology, and mathematical chemistry.

In this paper we take into account a simple, connected graph G having n vertices and m edges. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. A uv edge is the one that connects vertices u and v. The degree of a vertex v is the number of edges incident to it, and is denoted by $d_G(v)$.

The first and second Zagreb indices of graph G [9] are defined respectively as

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

The Zagreb indices were used in the structure property model [8, 11, 12,13]. The Forgotten topological index of a graph G is defined as [5]

$$F(G) = \sum_{u \in V(G)} (d_G(u))^3.$$

The Sombor index of graph G is defined as [6]

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

Basic results of Sombor index are obtained in [7]. Sombor index of chemical graphs are obtained in [1, 2]. Extremal values of the Sombor index of unicyclic graphs and bicyclic graphs are obtained in [3]. Sombor index of directed graphs is obtained in [4]. Sombor energy of graphs are considered in [15, 16]. For definitions and terminologies of graphs we refer the books [10, 18]. In this paper we obtain results for Sombor index of some graph valued functions of subdivision graphs.

Definition 1.1. *The subdivision graph $S(G)$ is the graph derived from G by inserting a new vertex into each edge of G.*

Definition 1.2. *The line graph of G is the graph $L(G)$ whose vertex set has a one-to-one correspondence with the edge set of G and two vertices in $L(G)$ are adjacent whenever the corresponding edges of G are adjacent.*

Definition 1.3. *The total graph of G, denoted by $T(G)$, is a graph with vertex set $V(T(G)) = V(G) \cup E(G)$ and two vertices in $T(G)$ are adjacent if and only if they are adjacent elements or they are incident elements in G.*

Definition 1.4. [17] *The semi-total point graph of G, denoted by $T_2(G)$, is a graph with vertex set $V(T_2(G)) = V(G) \cup E(G)$ and two vertices in $T_2(G)$ are adjacent if they are adjacent vertices in G or one is vertex and other is an edge, incident to it.*

Definition 1.5. [17] *The semi-total line graph of G, denoted by $T_1(G)$, is a graph with vertex set $V(T_1(G)) = V(G) \cup E(G)$ and two vertices in $T_1(G)$ are adjacent if they are adjacent edges in G or one is vertex and other is an edge, incident to it.*

Illustration 1.1. *Without loss of generality, referring to the Fig. 1, let e and f be adjacent edges at v in G. Let e' and e'' be the subdivision edges of an edge e in $S(G)$ and f' and f'' be the subdivision edges of an edge f in $S(G)$. Let u_e and u_f be the subdivision vertices on edges e and f respectively in $S(G)$.*

Degrees of all the vertices in total graph of the subdivision graph is given in [14]. Edge set $E(T(S(G)))$ of the total graph of the subdivision graph can be partitioned into sets E_1, E_2, E_3, E_4 and E_5, as

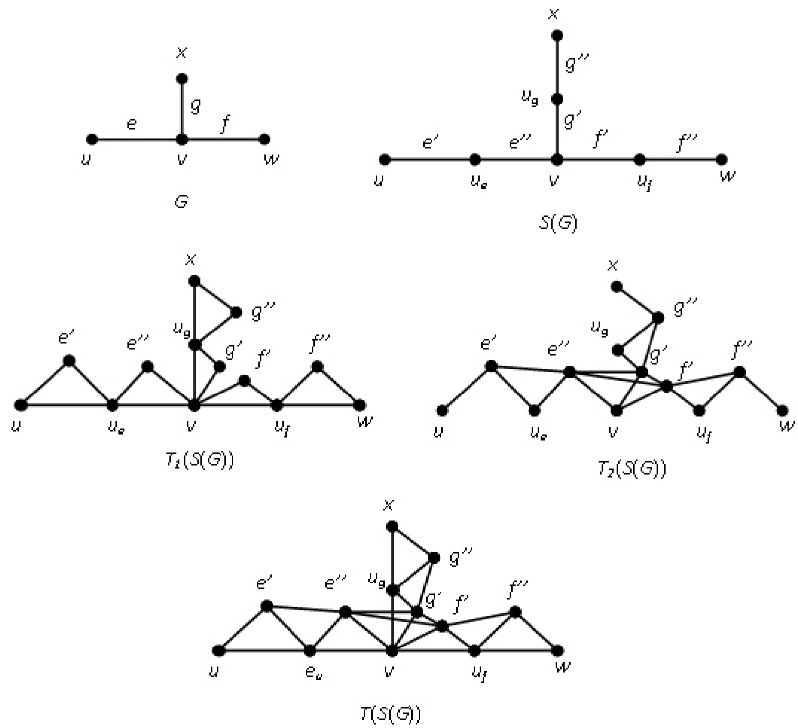

Figure 1: Graph G, $S(G)$, $T_2(S(G))$, $T_1(S(G))$ and $T(S(G))$.

$E_1 = \{uu_e \mid u \in V(G)$ and u_e is the subdivision vertex in $S(G)\}$,
$E_2 = \{ue' \mid u \in V(G)$ and e' is the subdivision edge in $S(G)\}$,
$E_3 = \{u_e e' \mid u_e$ is the subdivision vertex and e' is the subdivision edge in $S(G)\}$
$E_4 = \{e'e'' \mid e'$ and e'' are subdivision edges with common end vertex u_e in $S(G)\}$
and $E_5 = \{e''f' \mid e''$ and f' are subdivision edges in $S(G)$ with common end vertex v of $G\}$.

Easily we check that, $|E_1| = |E_2| = |E_3| = 2m$, $|E_4| = m$ and

$$|E_5| = \sum_{v \in V(G)} \frac{d_G(v)(d_G(v)-1)}{2} = -m + \frac{1}{2}M_1(G).$$

2. Main Results

Theorem 2.1. *Let G be a graph with n vertices and m edges. Then* $SO(T(S(G))) =$

$$2SO(S(G)) + \sum_{u \in V(G)} d_G(u) \left[\sqrt{5d_G(u)^2 + 4d_G(u) + 4} + \sqrt{d_G(u)^2 + 4d_G(u) + 20}\right] +$$

$$\sum_{uv \in E(G)} \sqrt{8 + d_G(u)^2 + d_G(v)^2 + 4(d_G(u) + d_G(v))} + \frac{1}{\sqrt{2}}[F(G) + M_1(G) - 4m]$$

Proof. By referring Fig. 1 and by degrees of the vertices in $T(S(G))$ we have $SO(T(S(G)))$

$$\begin{aligned}
&= \sum_{uv \in E(T(S(G)))} \sqrt{d_{T(S(G))}(u)^2 + d_{T(S(G))}(v)^2} \\
&= \sum_{uu_e \in E_1} \sqrt{d_{T(S(G))}(u)^2 + d_{T(S(G))}(u_e)^2} + \sum_{ue' \in E_2} \sqrt{d_{T(S(G))}(u)^2 + d_{T(S(G))}(e')^2} + \\
&\quad \sum_{u_e e' \in E_3} \sqrt{d_{T(S(G))}(u_e)^2 + d_{T(S(G))}(e')^2} + \sum_{e'e'' \in E_4} \sqrt{d_{T(S(G))}(e')^2 + d_{T(S(G))}(e'')^2} \\
&\quad + \sum_{e''f' \in E_5} \sqrt{d_{T(S(G))}(e'')^2 + d_{T(S(G))}(f')^2} \\
&= 2\sum_{u \in V(G)} \sqrt{d_G(u)^2 + 4} + \sum_{u \in V(G)} d_G(u)\sqrt{5d_G(u)^2 + 4d_G(u) + 4} \\
&\quad + \sum_{u \in V(G)} d_G(u)\sqrt{d_G(u)^2 + 4d_G(u) + 20} + \sqrt{2}\sum_{u \in V(G)} \binom{d_G(u)}{2}(2 + d_G(u))
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{uv\in E(G)}\sqrt{8+d_G(u)^2+d_G(v)^2+4(d_G(u)+d_G(v))}\\
=\;& 2SO(S(G))+\sum_{uv\in E(G)}\sqrt{8+d_G(u)^2+d_G(v)^2+4(d_G(u)+d_G(v))}\\
&+\sum_{u\in V(G)}d_G(u)\left[\sqrt{5d_G(u)^2+4d_G(u)+4}+\sqrt{d_G(u)^2+4d_G(u)+20}\right]\\
&+\frac{1}{\sqrt{2}}[F(G)+M_1(G)-4m].
\end{aligned}
$$

Theorem 2.2. *Let G be a graph with n vertices and m edges. Then,*

$$SO(T_2(S(G)))=2\sum_{u\in V(G)}d_G(u)\left[\sqrt{d_G(u)^2+4}+\sqrt{d_G(u)^2+1}+\sqrt{5}\right].$$

Proof. By referring Fig. 1 we see that the edge set $E(T_2(S(G)))=E_1\cup E_2\cup E_3$. Therefore

$$
\begin{aligned}
SO(T_2(S(G))) &= \sum_{uv\in E(T_2(S(G)))}\sqrt{d_{T_2(S(G))}(u)^2+d_{T_2(S(G))}(v)^2}\\
&= \sum_{uu_e\in E_1}\sqrt{d_{T_2(S(G))}(u)^2+d_{T_2(S(G))}(u_e)^2}+\sum_{ue'\in E_2}\sqrt{d_{T_2(S(G))}(u)^2+d_{T_2(S(G))}(e')^2}\\
&\quad+\sum_{u_ee'\in E_3}\sqrt{d_{T_2(S(G))}(u_e)^2+d_{T_2(S(G))}(e')^2}\\
&= \sum_{u\in V(G)}d_G(u)\sqrt{4d_G(u)^2+16}+\sum_{u\in V(G)}d_G(u)\sqrt{4d_G(u)^2+4}+\sum_{u\in V(G)}d_G(u)\sqrt{20}\\
&= 2\sum_{u\in V(G)}d_G(u)\left[\sqrt{d_G(u)^2+4}+\sqrt{d_G(u)^2+1}+\sqrt{5}\right].
\end{aligned}
$$

Theorem 2.3. *Let G be a graph with n vertices and m edges. Then,*

$$SO(T_1(S(G)))=\sum_{u\in V(G)}d_G(u)\left[\sqrt{2d_G(u)^2+4d_G(u)+4}+\sqrt{d_G(u)^2+4d_G(u)+8}\right]$$

$$+\sum_{uv\in E(G)}\sqrt{8+d_G(u)^2+d_G(v)^2+4(d_G(u)+d_G(v))}+\frac{1}{\sqrt{2}}[F(G)+M_1(G)-4m].$$

Proof. By referring Fig. 1 we see that the edge set $E(T_1(S(G))) = E_2 \cup E_3 \cup E_4 \cup E_5$. Therefore, we have $SO(T_1(S(G)))$

$$\begin{aligned}
&= \sum_{uv \in E(T_1(S(G))} \sqrt{d_{T_1(S(G))}(u)^2 + d_{T_1(S(G))}(v)^2} \\
&= \sum_{ue' \in E_2} \sqrt{d_{T_1(S(G))}(u)^2 + d_{T_1(S(G))}(e')^2} + \sum_{u_e e' \in E_3} \sqrt{d_{T_1(S(G))}(u_e)^2 + d_{T_1(S(G))}(e')^2} \\
&\quad + \sum_{e'e'' \in E_4} \sqrt{d_{T_1(S(G))}(e')^2 + d_{T_1(S(G))}(e'')^2} \\
&\quad + \sum_{e''f' \in E_5} \sqrt{d_{T_1(S(G))}(e'')^2 + d_{T_1(S(G))}(f')^2} \\
&= \sum_{u \in V(G)} d_G(u)\sqrt{d_G(u)^2 + (2 + d_G(u))^2} + \sum_{u \in V(G)} d_G(u)\sqrt{4 + (2 + d_G(u))^2} \\
&\quad + \sum_{uv \in E(G)} \sqrt{(2 + d_G(u))^2 + (2 + d_G(v))^2} + \sqrt{2} \sum_{u \in V(G)} \binom{d_G(u)}{2} (2 + d_G(u)) \\
&= \sum_{u \in V(G)} d_G(u) \left[\sqrt{2d_G(u)^2 + 4d_G(u) + 4} + \sqrt{d_G(u)^2 + 4d_G(u) + 8} \right] + \\
&\quad \sum_{uv \in E(G)} \sqrt{8 + d_G(u)^2 + d_G(v)^2 + 4(d_G(u) + d_G(v))} + \frac{[F(G) + M_1(G) - 4m]}{\sqrt{2}}.
\end{aligned}$$

Theorem 2.4. *Let G be a graph with n vertices and m edges. Then*

$$SO(L(S(G))) = SO(G) + \frac{1}{\sqrt{2}}[F(G) - M_1(G)].$$

Proof. Partition the edge set of line graph of subdivision graph into two sets, $E_1 = \{ef \mid e \sim u \text{ and } f \sim v \text{ in } S(G) \text{ where } uv \in E(G)\}$, and $E_2 = \{ef \mid e \sim u \text{ and } f \sim u \text{ in } S(G) \text{ where } u \in V(G)\}$. Also, $d_{L(S(G))}(e) = d_G(u)$ [13]. Hence, $SO(L(S(G)))$

$$\begin{aligned}
SO(L(S(G))) &= \sum_{ef \in E(L(S(G)))} \sqrt{(d_{L(S(G))}(e))^2 + (d_{L(S(G))}(f))^2} \\
&= \sum_{ef \in E_1} \sqrt{(d_{L(S(G))}(e))^2 + (d_{L(S(G))}(f))^2}
\end{aligned}$$

$$
\begin{aligned}
& + \sum_{ef \in E_2} \sqrt{(d_{L(S(G))}(e))^2 + (d_{L(S(G))}(f))^2} \\
= & \sum_{uv \in E(G)} \sqrt{(d_G(u))^2 + (d_G(v))^2} + \sum_{u \in V(G)} \sqrt{(d_G(u))^2 + (d_G(u))^2} \\
= & \; SO(G) + \sqrt{2} \sum_{u \in V(G)} \binom{d_G(u)}{2} d_G(u) \\
= & \; SO(G) + \frac{1}{\sqrt{2}} [F(G) - M_1(G)].
\end{aligned}
$$

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